Gravity-driven flow of Herschel-Bulkley fluid in a fracture and in a 2D porous medium

Vittorio Di Federicoa, Sandro Longob, Stuart E. Kingc, Luca Chiapponib, Diana Petrolob and Valentina Cirielloa

1. Implications of fluid rheology on flow in fractures and porous media

The behaviour of Herschel-Bulkley fluids flowing in a narrow channel (a fracture) has not been investigated to the same extent as flows in wide channels, and deserves an in-depth analysis due to the numerous practical applications of the process, such as polymer processing, heavy oil flow, gel cleanup in propped fractures, drilling processes.

2. Model description for free-surface flow in a narrow fracture

HB model for a shear thinning/thickening fluid with yield stress:

\[ f = (\gamma_0)^{2-1} + \gamma \gamma^1, \quad \gamma \geq \gamma_0 \]

\[ \eta = \frac{1}{\gamma} \]

\[ n = 1 \] shear-thinning

\[ n = 1 \] Bingam

\[ n > 1 \] shear-thickening

For free-surface flow through a narrow fracture of width \( W \):

The velocity \( \mathbf{v} = (v_x, v_y) \) in the fracture must satisfy

\[ \nabla \cdot \mathbf{v} = 0 \]

The velocity continuity of mass:

\[ \frac{\partial h}{\partial t} + \frac{v_x}{W} \frac{\partial h}{\partial x} + \frac{v_y}{W} \frac{\partial h}{\partial y} = 0 \]

Assuming a zero slip velocity, the evolution equation for \( h(x,y) \) is:

\[ \frac{\partial h}{\partial t} + \frac{v_x}{W} \frac{\partial h}{\partial x} + \frac{v_y}{W} \frac{\partial h}{\partial y} = 0 \]

where \( \gamma_0 \) is the critical stress. \( \gamma \) is a strain-like parameter; \( \gamma_0 \) is the yield stress of the fluid; \( n \) is the fluid behavior index.

3. Self-similar solution

For \( \gamma = \epsilon \gamma_0 \), a velocity scale given by \( (\gamma_0/\gamma)^{1/2} \), arises, and a self-similar solution of the form \( h = \epsilon h(x/\epsilon^{1/2}) \), with \( \epsilon \) small, yields:

\[ \frac{\partial h}{\partial t} + \frac{v_x}{W} \frac{\partial h}{\partial x} + \frac{v_y}{W} \frac{\partial h}{\partial y} = 0 \]

where \( \epsilon = (\gamma_0/\gamma)^{1/2} \) is the ratio between the two velocity scales. This system admits a simple solution, namely a linear profile for \( f(x) \).

Assuming a zero slip velocity, the evolution equation for \( h(x,y) \) is:

\[ \frac{\partial h}{\partial t} + \frac{v_x}{W} \frac{\partial h}{\partial x} + \frac{v_y}{W} \frac{\partial h}{\partial y} = 0 \]

where \( \gamma_0 \) is the critical stress. \( \gamma \) is a strain-like parameter; \( \gamma_0 \) is the yield stress of the fluid; \( n \) is the fluid behavior index.

References


