NON-NEWTONIAN FLOW IN A VARIABLE APERTURE FRACTURE: EFFECT OF FLUID RHEOLOGY

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KEY POINTS

- *A conceptual model is presented for non-Newtonian fluid flow in fractured media. The fluid is represented via a truncated power-law model, the fracture aperture via a stochastic model*
- *The fracture flowrate is derived for flow perpendicular to aperture variation as a function of the parameters describing the fluid rheology and the variability of the aperture field*
- *Adoption of the pure power law model leads to overestimation of the flowrate with respect to the truncated model, more so for large external pressure gradient and/or aperture variability*

1 INTRODUCTION

Hydraulic fracturing is largely used for optimal exploitation of oil, gas and thermal reservoirs. Non-Newtonian fluids are most frequently used in this type of operations (*Linkov*, 2014), and also in other application in reservoir engineering (*Ozdemirtas et al.,* 2010). Hence, it is important to model non-Newtonian flow in fractured media. A first step in this process is a detailed understanding of flow in a single fracture, as the space between fracture walls (termed fracture aperture) is typically spatially variable. Modeling this variability can be achieved with a deterministic (*Di Federico*, 1998) or stochastic (*Silliman*, 1989; *Di Federico*, 1997) approach, and the flowrate occurring in a single fracture under a given pressure gradient be determined as a function of the parameters describing the variability of the aperture field. From the flowrate, an equivalent aperture can then be derived (*Di Federico*, 1998; *Silliman*, 1989).

Another relevant issue is the rheological nature of the non-Newtonian fluid. Typically, power-law fluids governed by the four-parameter Carreau constitutive equation are employed (*Lavrov*, 2015); this rheological equation is, however, well approximated by the truncated power-law model, whose adoption is more suitable for numerical modeling of flow in variable aperture fractures (*Lavrov*, 2013). To this end, *Lavrov* (2015) derived the expressions for flow of a truncated power-law fluid between parallel walls under a constant pressure gradient.

This paper extends the adoption of the truncated power-law model to variable aperture fractures, with the aim of understanding the joint influence of rheology and aperture spatial variability. Section 2 summarizes results on flow of a truncated power-law fluid between parallel walls; Section 3 presents the general expression of the flowrate for flow perpendicular to aperture variation; Section 4 illustrates the case of a lognormal aperture variation, comparing results with those obtained for pure power-law fluids.

2 FLOW OF TRUNCATED POWER-LAW FLUID FLOW IN A CONSTANT APERTURE FRACTURE

Consider the flow of a shear-thinning non-Newtonian fluid in a fracture of width W and constant aperture b; the coordinate system is shown in Figures 1a-1b; the fracture walls are at $z=+b/2$ and $z=-b/2$. Suppose a uniform, positive pressure gradient $p_x=\left(p(0)-p(L)\right)/L$ is applied in the x direction. Assuming that $b<<W$, the velocity components in the y and z directions are zero, and the only nonzero velocity component v_x is solely a function of z. The fluid is described by the rheological truncated power-law model, reading, in the simple shear situation described above, $\tau=\mu_a\dot{\gamma}$, with τ shear stress and $\dot{\gamma}$ shear rate; the apparent viscosity μ_a is given by

$$\mu_a=\mu_0 \text{ for } \dot{\gamma}\le\dot{\gamma}_1;\ \mu_a=m\dot{\gamma}^{n-1} \text{ for } \dot{\gamma}_1<\dot{\gamma}<\dot{\gamma}_2;\ \mu_a=\mu_\infty \text{ for } \dot{\gamma}\ge\dot{\gamma}_2. \tag{1}$$

In eq. (1), depicted in Figure 1c, μ_0 is the viscosity at zero shear rate, μ_∞ is the limiting viscosity for $\dot{\gamma} \to \infty$, n and m are the rheological and consistency index, respectively, $\dot{\gamma}_1 = (m/\mu_0)^{1/(1-n)}$ is the lower shear rate at which the high viscosity cutoff μ_0 is introduced, and $\dot{\gamma}_2 = (m/\mu_\infty)^{1/(1-n)}$ is the higher shear rate at which the low viscosity cutoff μ_∞ is introduced. The above four-parameter model is identical to the pure power-law model of parameters n and m in the intermediate shear stress range $\dot{\gamma}_1 < \dot{\gamma} < \dot{\gamma}_2$, and overcomes the limitation of having $\mu_a \to \infty$ for $\dot{\gamma} \to 0$ and $\mu_a \to 0$ for $\dot{\gamma} \to \infty$. *Lavrov* (2015) showed that the truncated power-law model is practically indistinguishable, for practical purposes, from the Carreau model. He also derived the velocity field $v_x(z)$ and the flowrate per unit width $q_x = Q_x/W$ under a constant pressure gradient p_x. The flowrate expression can take three different expressions, given by

$$q_{xI}(b) = \frac{b^3 p_x}{12\mu_0} \text{ for } b < b_1 = \frac{2\mu_0 \dot{\gamma}_1}{p_x};$$
$$q_{xII}(b) = \frac{2(1-n)m^{3/(1-n)}}{3(2n+1)\mu_0^{(2n+1)/(1-n)} p_x^2} + \frac{2nb^{(2n+1)/n}}{2n+1}\left(\frac{p_x}{2^{n+1} m}\right)^{1/n} \text{ for } b_1 < b < b_2; \qquad (2a,b,c)$$
$$q_{xIII}(b) = \frac{b^3 p_x}{12\mu_\infty} - \frac{2(1-n)m^{3/(1-n)}}{3(2n+1)p_x^2}\left(\frac{1}{\mu_\infty^{(2n+1)/(1-n)}} - \frac{1}{\mu_0^{(2n+1)/(1-n)}}\right) \text{ for } b > b_2 = \frac{2m\dot{\gamma}_2^n}{p_x}.$$

According to eq. (2), three flow regimes (I = low shear rate regime, II = intermediate shear rate regime, and III = high shear rate regimes) are possible within the fracture, depending on the relationship between its aperture b and the two threshold apertures b_1 and b_2.

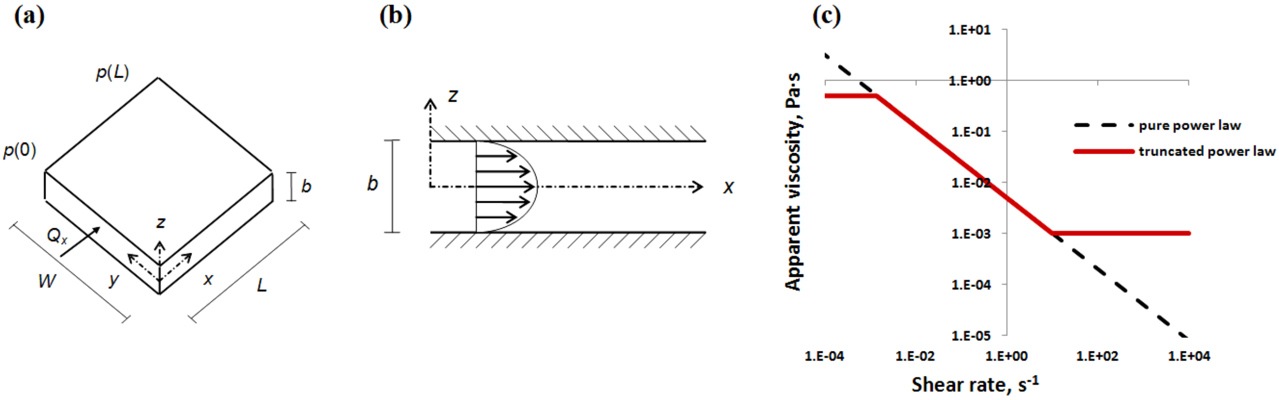

Figure 1. Panel (a) shows the fracture sketch with applied pressure gradient; panel (b) shows the fracture profile in the *x* direction; panel (c) depicts the apparent viscosity as a function of shear rate for the two models: truncated power-law and pure power-law.

3 FLOW IN A VARIABLE APERTURE FRACTURE

The fracture aperture is usually taken to vary as a two-dimensional, spatially homogeneous and correlated random field, with a probability density function *f*(*b*). The fracture dimensions are assumed to be much larger than the integral scale of the aperture autocovariance function. Then, under ergodicity, spatial averages and ensemble averages are interchangeable, and a single realization can be examined (*Silliman*, 1989; *Lavrov*, 2013). To derive an approximate expression for the equivalent aperture, one-dimensional flow can be considered in two limiting cases; in the first, the pressure gradient is transverse to the aperture variability; in the second, the pressure gradient is parallel to aperture variability. The equivalent aperture for flow in a 2-D aperture field is taken as the geometric average of the equivalent apertures derived for the two limit cases. This approach was used for Newtonian flow by *Silliman* (1989) to derive estimations of hydraulic and transport apertures, and by *Di Federico* (1997, 1998) to derive an estimate the hydraulic aperture for non-Newtonian flow under stochastic and deterministic aperture variation, respectively.

Here, we consider exclusively case 1 with flow parallel to constant aperture channels, i.e., transverse to aperture variation (Figure 2a); the applied pressure gradient is $p_x = (p(0) - p(L))/L$; the volumetric flux is obtained through the following procedure.

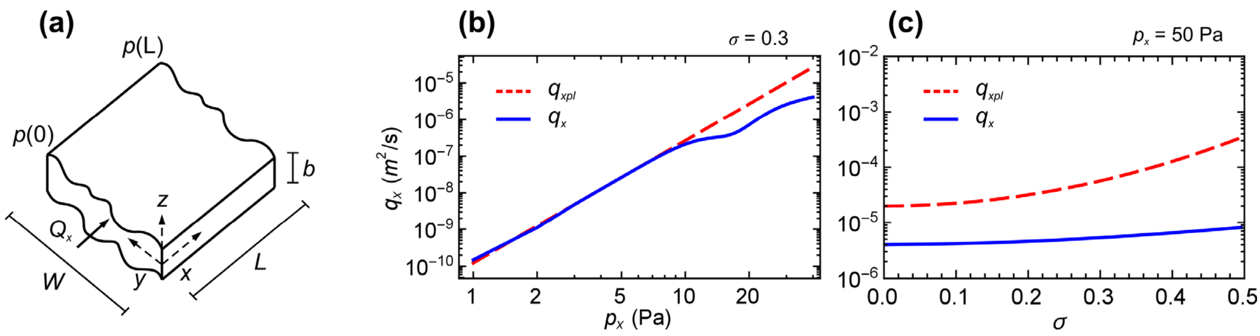

Figure 2. Panel (a) shows flow perpendicular to aperture variation described by the aperture density function $f(b)$; panel (b) depicts the flowrates for the truncated and pure power law case versus pressure gradient for fixed aperture variability; panel (c) does the same versus aperture variability for fixed gradient.

The fracture model is discretized into N neighboring channels, each having equal width and constant aperture b_i. Depending on the local aperture value, in each channel the flow regime is either I, or II, or III, and the corresponding flowrate per unit width is given either by eq. (2a), (2b), or (2c). The number of channels in each regime is N_I, N_{II}, N_{III}, respectively, and the total width of the channels in each regime is W_I, W_{II}, W_{III}, with $N = N_I + N_{II} + N_{III}$ and $W = W_I + W_{II} + W_{III}$; the i-th channel in each regime j ($j = 1,2,3$) has width $W_{ji} = W_j / N_j$. Assuming that the shear between neighboring channels and the drag against the connecting walls may be neglected, the total flowrate in the x direction is

$$Q_x = \sum_{i=1}^{N_I} q_I(b_i) W_{Ii} + \sum_{i=1}^{N_{II}} q_{II}(b_i) W_{IIi} + \sum_{i=1}^{N_{III}} q_{III}(b_i) W_{IIIi} . \quad (3)$$

Taking the limit as $N_j \to \infty$, the width of each channel tends to zero and the discrete aperture variation to a continuous one; then under ergodicity, and exploiting the previous relationships, eq. (3) gives for the flowrate per unit width in the x direction the expression

$$\begin{aligned} q_x = \frac{Q_x}{W} = P_I I_I \frac{p_x}{12\mu_0} + P_{II}\left[P_{II} \frac{2(1-n)m^{3/(1-n)}}{3(2n+1)\mu_0^{(2n+1)/(1-n)} p_x^2} + \frac{n}{2n+1} I_{II} \left(\frac{p_x}{2^{n+1} m} \right)^{1/n} \right] + \\ + P_{III}\left[I_{III} \frac{p_x}{12\mu_\infty} - P_{III} \frac{2(1-n)m^{3/(1-n)}}{3(2n+1)p_x^2} \left(\frac{1}{\mu_\infty^{(2n+1)/(1-n)}} - \frac{1}{\mu_0^{(2n+1)/(1-n)}} \right) \right] \end{aligned} , \quad (4)$$

$$I_I = \int_0^{b_1} b^3 f(b) db ; \; I_{II} = \int_{b_1}^{b_2} b^{(2n+1)/n} f(b) db ; \; I_{III} = \int_{b_2}^{\infty} b^3 f(b) db ; \; P_I = F(b_1) ; \; P_{II} = F(b_2) - F(b_1) ; \; P_{III} = 1 - F(b_2) , \quad (5)$$

where $f(b)$ and $F(b)$ are the pdf and cumulative distribution function of the aperture field, respectively.

4 Estimate of flowrate and discussion

A lognormal distribution is adopted for the aperture field, consistently with earlier work on flow and transport in variable aperture fractures (*Silliman*, 1989). Its probability distribution function is given by

$$f(b)=\frac{1}{b\sigma\sqrt{2\pi}}\exp\left[-\frac{\left(\ln b-\ln b_g\right)^2}{2\sigma^2}\right], \tag{6}$$

where $b_g = <b> \exp\left(-\sigma^2/2\right)$ is the geometric mean, $<b>$ the arithmetic mean, and σ^2 the variance of $\ln b$. Utilizing eqs. (4)-(5) with eq. (6) gives for the factors I_j and P_j (j = 1, 2, 3) the following expressions:

$$I_I=\frac{<b>^3}{2}\exp\left(3\sigma^2\right)\left[1+\operatorname{erf}\left(\frac{1}{\sqrt{2}\sigma}\left(\ln\frac{b_1}{<b>}-\frac{5\sigma^2}{2}\right)\right)\right],\quad \mathrm{I}_{II}=\frac{<b>^{(2n+1)/n}}{2}\exp\left(\frac{(2n+1)(n+1)}{2n^2}\sigma^2\right)\cdot$$
$$\cdot\left[\operatorname{erf}\left(\frac{1}{\sqrt{2}\sigma}\left(\ln\frac{b_2}{<b>}-\frac{(3n+2)\sigma^2}{2n}\right)\right)-\operatorname{erf}\left(\frac{1}{\sqrt{2}\sigma}\left(\ln\frac{b_1}{<b>}-\frac{(3n+2)\sigma^2}{2n}\right)\right)\right], \tag{7}$$
$$I_{III}=\frac{<b>^3}{2}\exp\left(3\sigma^2\right)\left[1-\operatorname{erf}\left(\frac{1}{\sqrt{2}\sigma}\left(\ln\frac{b_2}{<b>}-\frac{5\sigma^2}{2}\right)\right)\right],$$

$$P_I=\frac{1}{2}\left[1+\operatorname{erf}\left(\frac{1}{\sqrt{2}\sigma}\left(\ln\frac{b_1}{<b>}+\frac{\sigma^2}{2}\right)\right)\right],\; P_{II}=\frac{1}{2}\left[\operatorname{erf}\left(\frac{1}{\sqrt{2}\sigma}\left(\ln\frac{b_2}{<b>}+\frac{\sigma^2}{2}\right)\right)+\right.$$
$$\left.-\operatorname{erf}\left(\frac{1}{\sqrt{2}\sigma}\left(\ln\frac{b_1}{<b>}+\frac{\sigma^2}{2}\right)\right)\right];\; P_{III}=\frac{1}{2}\left[1-\operatorname{erf}\left(\frac{1}{\sqrt{2}\sigma}\left(\ln\frac{b_2}{<b>}+\frac{\sigma^2}{2}\right)\right)\right], \tag{8}$$

where erf(.) is the error function. The expression of the flowrate given by eq. (4) with eqs. (7-8) is compared with that of a pure power-law (*pl*) fluid of parameters m and n (*Di Federico*, 1998), i.e.

$$q_{xpl}=\frac{n}{2n+1}\left(\frac{p_x}{2^{n+1}m}\right)^{1/n}\langle b\rangle^{(2n+1)/n}\exp\left(\frac{(2n+1)(n+1)\sigma^2}{2n^2}\right). \tag{9}$$

Clearly, $q_x \to q_{xpl}$ for $\mu_0 \to \infty$ and $\mu_\infty \to 0$. Both flowrates q_x and q_{xpl} are plotted in Figures 2b and 2c for $\mu_0 = 0.5\,\mathrm{Pa\cdot s}$, $\mu_\infty = 0.001\,\mathrm{Pa\cdot s}$, $n = 0.3$ and $m = 0.005\ \mathrm{Pa\cdot s^n}$; Figure 2b depicts q_x and q_{xpl} versus p_x for fixed $\sigma = 0.3$; Figure 2c does so versus σ for fixed $p_x = 50\ \mathrm{Pa/m}$. It is seen that the flowrate for the truncated model is always decidedly smaller than that associated with the pure power-law, except for very small pressure gradients. The difference between the two increases as the external pressure gradient and aperture variability become larger. Hence, adoption of the pure power law model leads to a significant overestimation of the flowrate with respect to the more realistic truncated rheological model.

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