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## RESEARCH ARTICLE

### *Semi-global extension of maximally complex submanifolds*

Giuseppe Della Sala<sup>a</sup> and Alberto Saracco<sup>b\*</sup>

<sup>a</sup>*Facultät für Mathematik, Universität Wien, Nordbergstraße 15, A-1090 Wien, Austria;*

<sup>b</sup>*Dipartimento di Matematica, Università di Parma, viale G. Usberti 53/A, I-43124  
Parma, Italy*

( )

Let  $A$  be a domain of the boundary of a (weakly) pseudoconvex domain  $\Omega$  of  $n$  and  $M$  a smooth, closed, maximally complex submanifold of  $A$ . We find a subdomain  $E$  of  $n$ , depending only on  $\emptyset$  and  $A$ , and a complex variety  $W \subset E$  such that  $bW = M$  in  $E$ . Moreover, a generalization to analytic sets of depth at least 4 is given.

**Keywords:** boundary problem; pseudoconvexity; maximally complex submanifold; CR geometry

**AMS Subject Classification:** Primary 32V25, Secondary 32T15

## 1. Introduction

In the last fifty years, the *boundary problem*, i.e. the problem of characterizing real submanifolds which are boundaries of “something” analytic, has been widely treated.

The first result of this kind is due to Wermer [?]. We: compact real curves in  $n$  are boundaries of complex varieties if and only if they satisfy a global integral condition, the *moments condition*. For greater dimension the problem was solved, by Harvey and Lawson [?], HL, proving that an obviously necessary condition (*maximal complexity*) is also sufficient for compact manifolds in  $n$ . Later on, characterizations for closed (non necessarily compact) submanifolds in  $q$ -concave open subsets of  $\mathbb{P}^n$  were provided by Dolbeault-Henkin and Dihn in [?, ] DH, DH2, Di. A new approach to the problem in  $\mathbb{P}^n$  has been recently set forth by Harvey-Lawson [?, ] HL3, HL4, HL5, HL6.

Our goal is to drop the compactness hypothesis. The results in [?, ] DS deal with the global situation of submanifolds contained in the boundary of a special class of strongly pseudoconvex unbounded domains in  $n$ . In this paper we consider the boundary problem for complex analytic varieties in a “semi-global” setting.

The boundary problem has been considered in this context by Chirka [?, ] Ch, who proved a “relative” Harvey-Lawson result by showing that if a maximally complex manifold  $M$  is contained in  $n \setminus K$ , where  $K$  is a polynomially convex compact set, then there exists a solution  $W \subset n \setminus K$ .

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\*Corresponding author. Email: alberto.saracco@unipr.it

In our situation, we let  $\emptyset \subset^n$  be a smooth, (weakly) pseudoconvex open domain in  $^n$  with boundary  $b\Omega$ , and we let  $M$  be a smooth, maximally complex  $(2m+1)$ -dimensional real closed submanifold ( $m \geq 1$ ) of some open domain  $A \subset b\emptyset$  with  $K = b\emptyset \setminus A$ . We want to find a domain  $E$  of  $^n$ , not depending on  $M$ , and a complex subvariety  $W$  of  $E$  such that  $bW = M$  in  $E$  (possibly in the sense of currents).

In the paper we use a parameter version of Harvey-Lawson's theorem to construct a solution  $(E, W)$  to the problem above, where  $E$  can be characterized, roughly, in terms of the envelope of  $K$  with respect to the algebra of functions holomorphic in a neighbourhood of  $\emptyset$ . We refer to section ?? for a precise statement of the results, which in some ways echoes that of Lupaciolu on the extension of CR-functions (see [?, ]Theorem 2]Lu).

If  $A$  is not relatively compact, our theorem can be restated in terms of “principal divisors hull”, leading to a global result for unbounded strictly pseudoconvex domains, different from the results in [?, ]]DS. Indeed, this method of proof allows us to drop the Lupaciolu hypothesis in [?, ]]DS and extend the maximally complex submanifold to a domain, which can anyhow not be the whole of  $\emptyset$ . If the Lupaciolu hypothesis holds, then the domain of extension is in fact all of  $\Omega$ . So this result is actually a generalization of the one in [?, ]]DS.

The crucial question of the maximality of the domain  $E$  we construct is not answered; in some simple cases the domain is indeed maximal (see Example ??).

In the last section, by the same methods, the extension result is proved for analytic sets (see Theorem ??).

It worths noticing that in [?, ]]ST2 related results are obtained via a bump Lemma and cohomological methods. That approach may be generalized to complex spaces.

We wish to thank Giuseppe Tomassini for suggesting us the problem in the first place and for useful discussions.

## 2. Using the *gCOV* Class file

If the file `gCOV2e.cls` is not already in the appropriate system directory for  $\text{\LaTeX} 2_\epsilon$  files, either arrange for it to be put there, or copy it to your working folder. The *gCOV* document style is implemented as a complete document style, *not* a document style option. In order to use the *gCOV* style, replace ‘`article`’ by ‘`gCOV2e`’ in the `\documentclass` command at the beginning of your document:

```
\documentclass{article}
```

is replaced by

```
\documentclass{gCOV2e}
```

In general, the following standard document style options should *not* be used with the *gCOV* style:

- (1) `10pt`, `11pt`, `12pt`—unavailable;
- (2) `oneside` (no associated style file)—`oneside` is the default;
- (3) `leqno` and `titlepage`—should not be used;
- (4) `singlecolumn`—is not necessary as it is the default style.

### 2.1. Landscape pages

If a table or illustration is too wide to fit the standard measure, it must be turned, with its caption, through  $90^\circ$  anticlockwise. Landscape illustra-

tions and/or tables can be produced directly using the `gCOV2e` style file using `\usepackage{rotating}` after `\documentclass{gCOV2e}`. The following commands can be used to produce such pages.

```
\setcounter{figure}{2}
\begin{sidewaysfigure}
\centerline{\epsfbox{fig1.eps}}
\caption{This is an example of figure caption.}
\label{landfig}
\end{sidewaysfigure}

\setcounter{table}{0}
\begin{sidewaystable}
\tbl{The Largest Optical Telescopes.}
\begin{tabular}{@{}lccccl}
.
.
.
\end{tabular}\label{tab1}
\end{sidewaystable}
```

Before any float environment, use the `\setcounter` command as above to fix the numbering of the caption. Subsequent captions will then be automatically renumbered accordingly.

### 3. Additional features

In addition to all the standard  $\text{\LaTeX} 2_{\epsilon}$  design elements, *rCOV* style includes a separate command for specifying short versions of the authors' names and the journal title for running headlines on the left-hand (verso) and right-hand (recto) pages, respectively (see Section 4.4). In general, once you have used this additional `gCOV2e.cls` feature in your document, do not process it with a standard  $\text{\LaTeX} 2_{\epsilon}$  style file.

#### 3.1. Footnotes to article titles and authors' names

On the title page, the `\thanks` command may be used to produce a footnote to either the title or authors' names.

Footnote symbols should be used in the order: † (coded as `\dagger`), ‡ (`\ddagger`), § (`\S`), ¶ (`\P`), || (`\I`), †† (`\dagger\dagger`), ‡‡ (`\ddagger\ddagger`), §§ (`\S\S`), ¶¶ (`\P\P`), ||| (`\I\I\I`).

Note that footnotes to the text will automatically be assigned the superscript symbols 1, 2, 3,... by the Class file, beginning afresh on each page.<sup>1</sup>

The title, author(s) and affiliation(s) should be followed by the `\maketitle` command.

#### 3.2. Abstracts

At the beginning of your article, the title should be generated in the usual way using the `\maketitle` command. Immediately following the title you should include an

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<sup>1</sup>These symbols will be changed to the style of the journal by the typesetter during preparation of your proofs.

abstract. The abstract should be enclosed within an `abstract` environment. For example, the titles for this guide were produced by the following source code:

```
\title{{\itshape Complex Variables and Elliptic Equations: An %
International Journal}---\LaTeXe\ style guide for authors %
\break(Style 2 + References Style Q)}

\author{Taylor \& Francis Limited, 4 Park Square,
Milton Park, Abingdon, OX14 4RN, UK} \received{v3.2 released %
November 2007}

\maketitle
\begin{abstract}
This guide is for authors who are preparing papers for the Taylor \& %
Francis journal {\em Complex Variables and Elliptic Equations: An
International Journal} ({\it gCOV}\,) using the \LaTeXe\ document %
preparation system and the Class file {\tt gCOV2e.cls}, which is %
available via the journal homepage on the Taylor \& Francis website %
(see Section~\ref{FTP}). Authors planning to submit their papers in %
\LaTeXe\ are advised to use {\tt gCOV2e.cls} as early as possible %
in the creation of their files. \end{abstract}
```

(Please note that the percentage signs at the ends of lines that quote source code in this document are not part of the coding but have been inserted to achieve line wrapping at the appropriate points.)

### 3.3. Lists

The *gCOV* style provides numbered and unnumbered lists using the `enumerate` environment and bulleted lists using the `itemize` environment.

The enumerated list numbers each list item with arabic numerals:

- (1) first item
- (2) second item
- (3) third item

Alternative numbering styles can be achieved by inserting a redefinition of the number labelling command after the `\begin{enumerate}`. For example, the list

- (i) first item
- (ii) second item
- (iii) third item

was produced by:

```
\begin{enumerate}
\item[(i)] first item
\item[(ii)] second item
\item[(iii)] third item
\end{enumerate}
```

Unnumbered lists are also provided using the `enumerate` environment. For example,

First unnumbered indented item without label.  
 Second unnumbered item.  
 Third unnumbered item.

was produced by:

```
\begin{enumerate}
  \item[] First unnumbered indented item...
  \item[] Second unnumbered item.
  \item[] Third unnumbered item.
\end{enumerate}
```

Bulleted lists are provided using the `itemize` environment. For example,

- First bulleted item
- Second bulleted item
- Third bulleted item

was produced by:

```
\begin{itemize}
\item First bulleted item
\item Second bulleted item
\item Third bulleted item
\end{itemize}
```

#### 4. Some guidelines for using standard features

The following notes may help you achieve the best effects with the `gCOV2e Class` file.

##### 4.1. Sections

$\text{\LaTeX} 2_{\epsilon}$  provides five levels of section headings and they are all defined in the `gCOV2e Class` file:

- (1) `\section`
- (2) `\subsection`
- (3) `\subsubsection`
- (4) `\paragraph`
- (5) `\subparagraph`

Numbering is automatically generated for section, subsection, subsubsection and paragraph headings. If you need additional text styles in the headings, see the examples in Section 5.

##### 4.2. Illustrations (figures)

The *gCOV* style will cope with most positioning of your illustrations and you should not normally use the optional positional qualifiers of the `figure` environment, which would override these decisions. See ‘Instructions for Authors’ in the journal’s homepage on the Taylor & Francis website for how to submit artwork (<http://www.tandf.co.uk/journals/authors/gcovauth.asp>). Figure captions should be below the figure itself, therefore the `\caption` command should appear after the figure. For example, Figure 1 with caption and sub-captions is produced using the following commands:

(44)

Figure 1. Example of a two-part figure with individual sub-captions showing that all lines of figure captions range left.

Table 1. Radio-band beaming model parameters for FSRQs and BL Lacs.

Class <sup>a</sup>	$\gamma_1$	$\gamma_2^b$	$\langle\gamma\rangle$	$G$	$f$	$\theta_c$
BL Lacs	5	36	7	-4.0	$1.0 \times 10^{-2}$	$10^\circ$
FSRQs	5	40	11	-2.3	$0.5 \times 10^{-2}$	$14^\circ$

<sup>a</sup>This is not as accurate, owing to numerical error.

<sup>b</sup>An example table footnote to show the text turning over when a long footnote is inserted.

```

\begin{figure}
\begin{center}
\subfigure[] {
\resizebox*{5cm}{!}{\includegraphics{senu_gr1.eps}}}
\subfigure[] {
\resizebox*{5cm}{!}{\includegraphics{senu_gr2.eps}}}
\caption{\label{fig2} Example of a two-part figure with individual %
sub-captions showing that all lines of figure captions range left.}%
\label{sample-figure}
\end{center}
\end{figure}

```

The control sequences `\epsfig{}`, `\subfigure{}` and `\includegraphics{}` require `epsfig.sty`, `subfigure.sty` and `graphicx.sty`. These are called by the Class file `gCOV2e.cls` and are included with the LaTeX package for this journal for convenience.

To ensure that figures are correctly numbered automatically, the `\label{}` command should be inserted just after `\caption{}`

### 4.3. Tables

The *gCOV* style will cope with most positioning of your tables and you should not normally use the optional positional qualifiers of the `table` environment, which would override these decisions. The table caption appears above the body of the table in *gCOV* style, therefore the `\tbl` command should appear before the body of the table.

The `tabular` environment can be used to produce tables with single thick and thin horizontal rules, which are allowed, if desired. Thick rules should be used at the head and foot only and thin rules elsewhere.

Commands to redefine quantities such as `\arraystretch` should be omitted. For example, table 1 is produced using the following commands. Note that `\rm` will produce a roman character in math mode. There are also `\bf` and `\it`, which produce bold face and text italic in math mode.

```

\begin{table}
\tbl{Radio-band beaming model parameters
for {FSRQs and BL Lacs.}}
{\begin{tabular}{@{}lcccccc}\toprule
Class$\sim{\rm a}$}

```

```

& $\gamma _1$ & $\gamma _2$^{\rm b}$
& $\angle \gamma \angle $
& $G$ & $f$ & $\theta _{c}$ $ \backslash
\colrule
BL Lacs &5 & 36 & 7 & $-4.0$
& $1.0\times 10^{-2}$ & $10^{\circ}$ \backslash
FSRQs & 5 & 40 & 11 & $-2.3$
& $0.5\times 10^{-2}$ & $14^{\circ}$ \backslash
\botrule
\end{tabular}}
\tabnote{$^{\rm a}$}This is not as accurate, owing to
numerical error.}
\tabnote{$^{\rm b}$}An example table footnote to show the
text turning over when a long footnote is inserted.}%
\label{symbols}
\end{table}

```

To ensure that tables are correctly numbered automatically, the `\label{}` command should be inserted just before `\end{table}`.

#### 4.4. Running headlines

In *gCOV* style, the authors' names and the shortened title of the journal are used throughout the article as running headlines at the top of alternate pages. An abbreviated list of authors' names in italic format appears on even-numbered pages (versos)—e.g. '*J. Smith and P. Jones*', or '*J. Smith et al.*' for three or more authors—and the abbreviated journal title in italic format is used on odd-numbered pages (rectos). To achieve this, the `\markboth` command is used. The running headlines for this guide were produced using the following code: `\markboth{Taylor & Francis and I.T. Consultant}{Complex Variables and Elliptic Equations}`. The `\pagestyle` and `\thispagestyle` commands should *not* be used.

#### 4.5. Maths environments

The *gCOV* style provides for the following maths environments.

**Lemma 4.1:** *More recent algorithms for solving the semidefinite programming relaxation are particularly efficient, because they explore the structure of the MAX-CUT.*

**Theorem 4.2:** *More recent algorithms for solving the semidefinite programming relaxation are particularly efficient, because they explore the structure of the MAX-CUT.*

**Corollary 4.3:** *More recent algorithms for solving the semidefinite programming relaxation are particularly efficient, because they explore the structure of the MAX-CUT.*

**Proposition 4.4:** *More recent algorithms for solving the semidefinite programming relaxation are particularly efficient, because they explore the structure of the MAX-CUT.*

**Proof:** More recent algorithms for solving the semidefinite programming relaxation are particularly efficient, because they explore the structure of the MAX-



CUT. □

**Remark 1:** More recent algorithms for solving the semidefinite programming relaxation are particularly efficient, because they explore the structure of the MAX-CUT problem.

**Algorithm 1:** More recent algorithms for solving the semidefinite programming relaxation are particularly efficient, because they explore the structure of the MAX-CUT problem.

These were produced by:

```
\begin{lemma}
More recent algorithms for solving the semidefinite
programming relaxation are particularly efficient,
because they explore the structure of the MAX-CUT.
\end{lemma}
```

```
\begin{theorem}
...
...
\end{theorem}
```

```
\begin{corollary}
...
...
\end{corollary}
```

```
\begin{proposition}
...
...
\end{proposition}
```

```
\begin{proof}
...
...
\end{proof}
```

```
\begin{remark}
...
...
\end{remark}
```

```
\begin{algorithm}
...
...
\end{algorithm}
```

## 4.6. Typesetting mathematics

### 4.6.1. Displayed mathematics

The *gCOV* style will set displayed mathematics centred on the measure without equation numbers, provided that you use the  $\text{\LaTeX} 2_{\epsilon}$  standard control sequences

open (`\[`) and close (`\]`) square brackets as delimiters. The equation

$$\sum_{i=1}^p \lambda_i = \text{trace}(\mathbf{S}) \quad i \in \mathbb{R}$$

was typeset in the *gCOV* style using the commands

```
\[
  \sum_{i=1}^p \lambda_i = {\rm trace}({\texttrm{\bf S}})\qqquad
  i\in {\mathbb R}
\].
```

For those of your equations that you wish to be automatically numbered sequentially throughout the text, use the `equation` environment, e.g.

$$\sum_{i=1}^p \lambda_i = \text{trace}(\mathbf{S}) \quad i \in \mathbb{R} \tag{1}$$

was typeset using the commands

```
\begin{equation}
  \sum_{i=1}^p \lambda_i = {\rm trace}({\texttrm{\bf S}})\quad
  i\in {\mathbb R}
\end{equation}
```

Part numbers for sets of equations may be generated using the `subequations` environment, e.g.

$$\varepsilon \rho w_{tt}(s, t) = N[w_s(s, t), w_{st}(s, t)]_s, \tag{2a}$$

$$w_{tt}(1, t) + N[w_s(1, t), w_{st}(1, t)] = 0, \tag{2b}$$

which was generated using the control sequences

```
\begin{subequations} \label{subeqnexample}
\begin{equation}
  \varepsilon \rho w_{tt}(s, t)
  =
  N[w_s(s, t), w_{st}(s, t)]_s,
  \label{subeqnpart}
\end{equation}
\begin{equation}
  w_{tt}(1, t) + N[w_s(1, t), w_{st}(1, t)] = 0,
\end{equation}
\end{subequations}
```

This is made possible by the package `subeqn`, which is called by the `Class` file. If you put the `\label{}` just after the `\begin{subequations}` line, references will be to the collection of equations, ‘(2)’ in the example above. Or, like the example code above, you can reference each equation individually—e.g. ‘(2a)’.

#### 4.6.2. Bold math italic symbols

To get bold math italic you can use `\bm`, which works for all sizes, e.g.

```
\sffamily
\begin{equation}
  {\rm d}(\bm s_{t_{\bm u}}) = \langle \bm \alpha(\textbf L) \%
  [RM(\bm X_y + \bm s_t) - RM(\bm x_y)]^2 \rangle
\end{equation}
\normalfont
```

produces

$$d(s_{t_u}) = \langle \alpha(\mathbf{L})[RM(\mathbf{X}_y + \mathbf{s}_t) - RM(x_y)]^2 \rangle \quad (3)$$

Note that subscript, superscript, subscript to subscript, etc. sizes will take care of themselves and are italic, not bold, unless coded individually. `\bm` produces the same effect as `\boldmath`. `\sffamily...\normalfont` allows upright sans serif fonts to be created in math mode by using the control sequence ‘`\sf`’.

#### 4.6.3. Bold Greek

Bold lowercase as well as uppercase Greek characters can be obtained by `\bm \gamma`, which gives  $\gamma$ , and `\bm \Gamma`, which gives  $\Gamma$ .

#### 4.6.4. Upright lowercase Greek characters and the upright partial derivative sign

Upright lowercase Greek characters can be obtained with the Class file by inserting the letter ‘u’ in the control code for the character, e.g. `\umu` and `\upi` produce  $\mu$  (used, for example, in the symbol for the unit microns— $\mu\text{m}$ ) and  $\pi$  (the ratio of the circumference to the diameter of a circle). Similarly, the control code for the upright partial derivative  $\partial$  is `\upartial`.

#### 4.7. Acknowledgements

An unnumbered section, e.g. `\section*{Acknowledgement(s)}`, should be used for thanks, grant details, etc. and placed before any Notes or References sections.

#### 4.8. Notes

An unnumbered section, e.g. `\section*{Note(s)}`, may be inserted after any Acknowledgements and before any References section.

#### 4.9. Appendices

Appendices should be set after the references, beginning with the command `\appendices` followed by the command `\section` for each appendix title, e.g.

```
\appendices
\section{This is the title of the first appendix}
\section{This is the title of the second appendix}
```

produces

Appendix A. This is the title of the first appendix  
 Appendix B. This is the title of the second appendix

Subsections, equations, theorems, figures, tables, etc. within appendices will then be automatically numbered as appropriate.

## 4.10. References

### 4.10.1. References cited in the text

References cited in the text should be quoted by number (e.g. [1], [2,4,10], [21–55], not [21]–[55]). References should be listed in the `\thebibliography` environment in the order in which they first appear within the main text. Each bibliographical entry has a key, which is assigned by the author and used to refer to that entry in the text. In this document, the key `neu83` in the citation form `\cite{neu83}` produces ‘[1]’, and the keys `ed84` and `aiex02` in the citation form `\cite{ed84,aiex00}` produce ‘[2, 3]’. The citation for a range of bibliographic entries (e.g. ‘[?, ]neu83,ed84,aiex00,aiex02,glov00,glov86,mtw73,hk96,fzf88,fwp88’) will automatically be produced by `\cite{neu83,ed84,aiex00,glov00,glov86,mtw73,hk96,fzf88,fwp88}`. Optional notes may be included at the end of a citation by the use of square brackets, e.g. `\cite[see][and references therein]{fzf88}` produces ‘[?, see]and references therein[fzf88]’.

### 4.10.2. The list of references

The following listing shows some references prepared in the style of the journal; note that references with more than ten authors begin with the first-named author’s initials and surname followed by ‘et al.’; references having the same author(s) are listed chronologically, beginning with the earliest, using three em-dashes instead of repeating the author list in the second and subsequent references:

## References

10

- [1] M. Neumann, *Parallel GRASP with path-relinking for job shop scheduling*, Mol. Phys. 50 (1983), pp. 841–843.
- [2] D.M.F. Edwards and I.R. McDonald, *Positive bases in numerical optimization*, Comput. Optim. Appl. 21 (1984), pp. 169–175.
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- [11] T.G. Golda, P.D. Hough, and G. Gay, APPSPACK (Asynchronous parallel pattern search package); software available at <http://software.sandia.gov/appspack>.

This list was produced by:

\begin{thebibliography}{10}

\bibitem[1]{neu83}%1

M. Neumann, {\em Parallel GRASP with path-relinking for job shop % scheduling}, Mol. Phys. 50 (1983), pp. 841--843.

\bibitem[2]{ed84}%2

D.M.F. Edwards and I.R. McDonald, {\em Positive bases in numerical % optimization}, Comput. Optim. Appl. 21 (1984), pp. 169--175.

\bibitem[3]{aiex00}%3

R.M. Aiex, I.F. Pierce, G. Donizetti, C.M. {\v{on}Weber}, G. Bizet, % C.P.E. Bach, R. Strauss, L. {\v{an}Beethoven}, W.A. Mozart, and P. % Dukas, {\em Computing tools for modelling orchestral performance}, % Tech. Rep. DAMTP 2000/NA10, Department of Applied Mathematics and % Theoretical Physics, University of Cambridge, Cambridge, UK, 2000.

\bibitem[4]{aiex02}%4

R.M. Aiex et al., {\em Conjectured statistics for the %  $\{q,t\}$ -Catalan numbers}, to appear in Advances in Math. (preprint % 2002), Available at <http://www.math.caltech.edu/~aiex>.

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F. Glover, {\it Multi-start and strategic oscillation methods---% principles to exploit adaptive memory}}, in {\it Computing Tools % for Modeling, Optimization and Simulation: Interfaces in Computer % Science and Operations Research}, M. Laguna and J.L. Gonz\~ales- % Velarde, eds., 2nd ed., Kluwer Academic, Boston, MA, 2000, pp. 1--24.

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H. Kern, {\em The resurgent Japanese economy and a Japan--United % States free trade agreement}}, in {\em 4th International Conference % on the Restructuring of the Economic and Political System in Japan % and Europe}}, Milan, Italy, 21--25 May 1996, World Scientific, % Singapore, 1997, pp. 147--156.

\bibitem[9]{fzf88}%9

F. French, {\em English title of a chapter in the translation of a % book in a foreign language}}, in {\itshape Title of a Book in Another %

Language (Quoted in that Language)}, P. Smith (Transl.), Dover, New %  
York, 1988 (original work  
published 1923).

`\bibitem[10]{fwp88}%10`

F.W. Patel, `{\itshape Title of a Book}`, Monographs on Technical %  
Aspects Vol. II, Dover, New York, 1988.

`\bibitem[11]{GHGsoft}%11`

T.G. Golda, P.D. Hough, and G. Gay, APPSPACK (Asynchronous parallel %  
pattern search package); software available at <http://software.sandia.gov/appspack>.

`\end{thebibliography}`

Each entry takes the form:

`\bibitem{key} Bibliography entry`

where `key` is the tag that is to be used as an argument for the `\cite{}` command. `Bibliography entry` should be the material that is to appear in the bibliography, suitably formatted.

Instead of including ‘thebibliography’ environment in the main source file of their article, authors may include the lines

`\bibliographystyle{gCOV}`

`\bibliography{gCOVguide}`

where the references list should appear, where `gCOV.bst` is the BiBTeX style file for this journal and `gCOVguide.bib` is the database of bibliographic details for the references section (both included with the `gCOV` LaTeX style guide package). `gCOVguide.bib` can be used as a template for creating your database, which can be used with any of your future papers. The  $\text{\LaTeX}$  2 $\epsilon$  source file of a particular paper will extract from the `.bib` file only those references that are cited in that paper and listed in the references section of it. Thus

`\bibliographystyle{gCOV}`

`\bibliography{gCOVguide}`

produces:

## References

0

Note that only ten of the eleven `bibitem`s in the `.bib` file have appeared in the above references list because these are the only ten cited in the text of this guide.

### 4.11. *gCOV* macros

Table 2 gives a list of macros for use with *gCOV*. The list displays each macro’s code and a description/demonstration of its function.

Table 2. *gCOV* macros.

<code>\markboth{short author(s) list}{short journal title}</code>	short author(s) list and short journal title used in running heads (verso/recto, resp.)
<code>\thanks{title-page footnote to article title or author}</code>	e.g. ‘Corresponding author. E-mail: A.N. Author@uiowa.edu’
<code>\begin{abstract}...\end{abstract}</code>	for abstract on titlepage
<code>\bm{math and symbols}</code>	bold italic <i><b>math and symbols</b></i>
<code>\bi{text}</code>	bold italic <i><b>text</b></i>
<code>\sf{text or upright symbols in math mode}</code>	sans serif text or upright symbols in math mode

## 5. Example of a section heading with SMALL CAPS, lowercase, *italic*, and bold Greek such as $\kappa$

The following code shows how to achieve this section head:

```
\section{Example of section heading with\\*
  {\fontencoding{T1}\scshape\lowercase{small caps}},
  \lowercase{lowercase}, {\bi italic},
  and bold\\* Greek such as
  ${\bm\kappa}$}\label{headings}
```

## 6. *gCOV* journal style

The notes given here relate to common style errors found in *gCOV* manuscripts, but are *not* intended to be exhaustive.

### 6.1. *Punctuation*

When deciding where to add commas, it may be helpful to read through the sentence and note where the natural ‘pauses’ occur. The needs of readers for whom English is not a first language should be borne in mind when punctuating long sentences. For example, consider the following sentence as it appeared in *gCOV*: ‘When we do not limit ourselves by constraints arising from the choice of an initial fluctuation spectrum, structures in an open universe, including the peculiar velocity structure, can be reproduced in a flat Lemaître universe for a large part of their evolution.’ Now consider the same sentence without commas: ‘When we do not limit ourselves by constraints arising from the choice of an initial fluctuation spectrum structures in an open universe including the peculiar velocity structure can be reproduced in a flat Lemaître universe for a large part of their evolution.’

### 6.2. *Spelling*

Please use British spelling—e.g. centre not center, labelled not labeled. The following style regarding -ise, -yse and -ize spellings is used: -ise—devise, surprise, comprise, revise, exercise; -yse—analyse; -ize: recognize, criticize, minimize, emphasize, organize.

### 6.3. Hyphens, n-rules, m-rules and minus signs

- (i) Hyphens (one dash in  $\text{T}_{\text{E}}\text{X}/\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X } 2_{\epsilon}$ ). *gCOV* uses hyphens for compound adjectives (e.g. low-density gas, least-squares fit, two-component model) but not for complex units or ranges, which could become cumbersome (e.g.  $15 \text{ km s}^{-1}$  feature,  $100\text{--}200 \text{ }\mu\text{m}$  observations).
- (ii) n-rules (two dashes in  $\text{T}_{\text{E}}\text{X}/\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X } 2_{\epsilon}$ ). These are used (a) to denote a range (e.g.  $1.6\text{--}2.2 \text{ }\mu\text{m}$ ); and (b) to denote the joining of two words of equal standing (e.g. Kolmogorov–Smirnov test, Herbig–Haro object).
- (iii) The m-rule (three dashes in  $\text{T}_{\text{E}}\text{X}/\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X } 2_{\epsilon}$ ) is used in *gCOV* as an alternative to parentheses (e.g. ‘the results—assuming no temperature gradient—are indicative of ...’).
- (iv) The minus sign (one dash in  $\text{T}_{\text{E}}\text{X}/\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X } 2_{\epsilon}$ ) is produced automatically in math mode by use of a single dash, e.g.

$$y_i \in \{-1, 1\} \quad \forall i \in V \quad (4)$$

where  $|-V| = A^2 + B^2$

is produced by

```
\begin{equation}
y_{\{i\}} \in \{-1, 1\} \quad \forall i \in V
\end{equation}
\noindent where  $|-V|=A^2+B^2$ 
```

### 6.4. References

It is important to use the correct reference style, details of which can be found in Section 4.10 above.

### 6.5. Maths fonts

Scalar variables should be mediumface italic (e.g. *s* for speed); vectors should be bold italic (e.g. ***v*** for velocity); matrices should be bold roman (upright) (e.g. **A**), and tensors should be bold upright sans serif (e.g. **L**). Differential d, partial differential  $\partial$ , complex i, exponential e, superscript T for ‘transpose’, sin, cos, tan, log, etc., should all be roman. Openface, or ‘blackboard’, fonts can be used, for example, for the integers  $\mathbb{Z}$  and the reals  $\mathbb{R}$ . Sub/superscripts that are physical variables should be italic, while those that are labels should be roman (e.g.  $C_p$ ,  $T_{\text{eff}}$ ). Displayed equations should have end-of-line punctuation appropriate to the running text sentence of which they form a part.

## 7. Troubleshooting

Authors may from time to time encounter problems with the preparation of their papers in  $\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X } 2_{\epsilon}$ . The appropriate action to take will depend on the nature of the problem – the following is intended to act as a guide.

- (i) If the problem is with  $\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X } 2_{\epsilon}$  itself, rather than with the actual macros, please refer to the appropriate handbooks for initial advice.<sup>1</sup> If the solution

---

<sup>1</sup> $\text{T}_{\text{E}}\text{X}$ : Knuth, D., 1986, *The  $\text{T}_{\text{E}}\text{X}$  book* (New York: Addison–Wesley);  $\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X } 2_{\epsilon}$ : Lamport, L., 1985,  *$\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X } 2_{\epsilon}$*



cannot be found, and you suspect that the problem lies with the macros, then please contact Taylor & Francis ([latex.helpdesk@tandf.co.uk](mailto:latex.helpdesk@tandf.co.uk)).

- (ii) Problems with page make-up (e.g. large spaces between paragraphs, or under headings or figures; uneven columns; figures/tables appearing out of order): please do *not* attempt to remedy these yourself using ‘hard’ page make-up commands – the typesetter will correct such problems. (You may, if you wish, draw attention to particular problems when submitting the final version of your paper.)
- (iii) If a required font is not available at your site, allow T<sub>E</sub>X to substitute the font and specify which font you require in the covering letter accompanying your file(s).

### 7.1. *Fixes for coding problems*

This guide has been designed to minimize the need for user-defined macros to create special symbols. Authors are urged, wherever possible, to use the following coding rather than to create their own. This will minimize the danger of author-defined macros being accidentally ‘over-ridden’ when the paper is typeset in Times (see Section 4.6, ‘Typesetting mathematics’ above). In cases where it is essential to create your own macros, these should be displayed in the preamble of the source file before `\begin{document}`.

- (i) Fonts in section headings and paper titles. The following are examples of styles that sometimes prove difficult to code.

#### *Paper titles:*

##### **Generalized Flory theory at $\delta > 50^\circ$**

is produced by

```
\title{Generalized Flory theory at
      ${\bm\delta} > {\bfseries 50}^\circ$}
```

##### **Ion–ion correlations in H II regions**

is produced by

```
\title{Ion--ion correlations in H\,{\sc ii} regions}
```

- (ii) n-rules, m-rules, hyphens and minus signs (see Section 6.3 for correct usage). To create the correct symbols in the sentence

The high-resolution observations were made along a line at an angle of  $-15^\circ$  (East from North) from the axis of the jet—which runs North–South

you would use the following code:

```
The high-resolution observations were made along a line at an
angle of $-15^\circ$ (East from North) from the axis of the
jet---which runs North--South
```

- (iii) Fonts in superscripts and subscripts. Subscripts and superscripts will automatically come out in the correct font and size in a math environment (e.g.

enclosed by ‘\$’ delimiters in running text or within  $\backslash[ \dots \backslash]$  or the ‘equation’ environment for displayed equations). You can create the output  $\mathbf{k}_x$  by typing  $\backslash\bm{k}_x$ . If the subscripts or superscripts need to be other than italic, they should be coded individually—see (vi) below.

- (iv) Calligraphic letters (uppercase only). Normal calligraphic can be produced with  $\backslashcal$  as usual (in math mode).
- (v) Automatic scaling of brackets. The codes  $\backslashleft$  and  $\backslashright$  should be used to scale brackets automatically to fit the equation being set. For example, to get

$$v = x \left( \frac{N+2}{N} \right)$$

use the code

```
\[
v = x \left( \frac{N+2}{N} \right)
\]
```

- (vi) Roman font in equations. It is often necessary to make some symbols roman in an equation (e.g. units, non-variable subscripts). For example, to get the following output:

$$\sigma \simeq (r/13 \text{ h}^{-1} \text{ Mpc})^{-0.9}, \quad \omega = \frac{N - N_s}{N_R}$$

you should use:

```
\[
\sigma \simeq (r/13\text{h}^{-1}
~{\rm Mpc})^{-0.9}, \quad \omega = \frac{N-N_{{\rm s}}}{{N_{{\rm R}}}}
\]
```

## 8. Obtaining the gCOV2e Class file

### 8.1. Via the Taylor & Francis website

This Guide for Authors and the gCOV2e.cls Class file may be obtained via the Instructions for Authors on the Taylor & Francis homepage for the journal (<http://www.tandf.co.uk/journals/titles/17476933.asp>).

Please note that the Class file calls up the following open-source LaTeX packages, which will, for convenience, unpack with the downloaded Guide for Authors and Class file: amsbsy.sty, amsfons.sty, amsmath.sty, amssymb.sty, enumerate.sty, epsfig.sty, graphicx.sty, mathbbol.sty, natbib.sty, rotating.sty, subfigure.sty.

### 8.2. Via e-mail

This Guide for Authors, the Class file and the associated open-source LaTeX packages are also available by e-mail. Requests should be addressed to [latex.helpdesk@tandf.co.uk](mailto:latex.helpdesk@tandf.co.uk) clearly stating for which journal you require the Guide for Authors and/or Class file.