EX 2

$$F = \begin{bmatrix} 1,5 & 0 & 0 \\ 0 & 0,3 & 0 \\ 0 & 0 & 0,3 \end{bmatrix}$$

Green Lagrange deformation tensor E:

$$E = \frac{1}{2}(C - I) = \frac{1}{2}(F^T F - I)$$

$$J = \det F = 0,135$$

$$F^T F = \begin{bmatrix} 2,25 & 0 & 0 \\ 0 & 0,09 & 0 \\ 0 & 0 & 0,09 \end{bmatrix} \qquad E = \frac{1}{2}\left\{\begin{bmatrix} 2,25 & 0 & 0 \\ 0 & 0,09 & 0 \\ 0 & 0 & 0,09 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right\} =$$

$$= \frac{1}{2}\begin{bmatrix} 1,25 & 0 & 0 \\ 0 & -0,91 & 0 \\ 0 & 0 & -0,91 \end{bmatrix} ; \quad \mathrm{tr}\,E = -0,285$$

2nd Piola-Kirchhoff stress tensor

$$S = \mathbb{C}E = I\lambda\,\mathrm{tr}E + 2\mu E = -0,285 \cdot 1,5 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 2 \cdot 0,9 \cdot \begin{bmatrix} 0,625 & 0 & 0 \\ 0 & -0,455 & 0 \\ 0 & 0 & -0,455 \end{bmatrix}$$

$$S = \begin{bmatrix} -0,4275 & 0 & 0 \\ 0 & -0,4275 & 0 \\ 0 & 0 & -0,4275 \end{bmatrix} + \begin{bmatrix} 1,125 & 0 & 0 \\ 0 & -0,819 & 0 \\ 0 & 0 & -0,819 \end{bmatrix} = \begin{bmatrix} 0,6975 & 0 & 0 \\ 0 & -1,2465 & 0 \\ 0 & 0 & -1,2465 \end{bmatrix} \text{MPa}$$

Cauchy stress tensor

$$P = FS = \begin{bmatrix} 1,5 & 0 & 0 \\ 0 & 0,3 & 0 \\ 0 & 0 & 0,3 \end{bmatrix} \cdot \begin{bmatrix} 0,6975 & 0 & 0 \\ 0 & -1,2465 & 0 \\ 0 & 0 & -1,2465 \end{bmatrix} = \begin{bmatrix} 1,04625 & 0 & 0 \\ 0 & -0,3739 & 0 \\ 0 & 0 & -0,3739 \end{bmatrix}$$

1st Piola

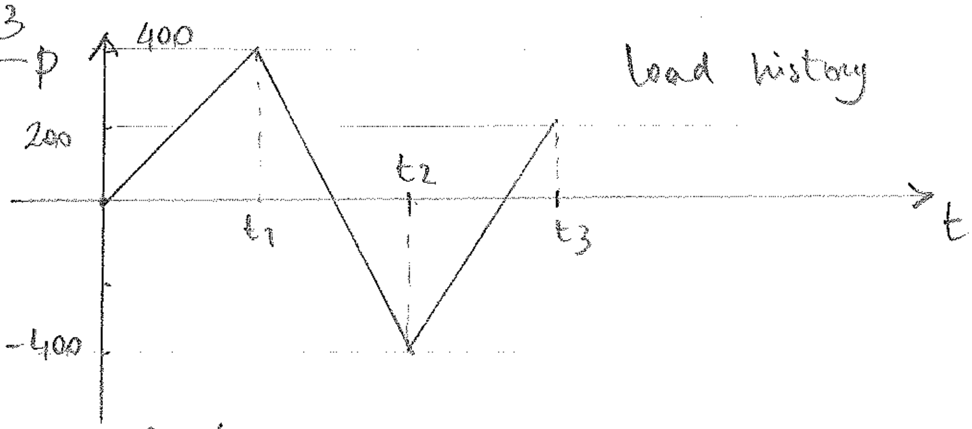
$$\sigma = J^{-1} P F^T = \frac{1}{0,135}\begin{bmatrix} 1,0462 & 0 & 0 \\ 0 & -0,3739 & 0 \\ 0 & 0 & -0,3739 \end{bmatrix} \cdot \begin{bmatrix} 1,5 & 0 & 0 \\ 0 & 0,3 & 0 \\ 0 & 0 & 0,3 \end{bmatrix} =$$

Cauchy stress

$$= 7,407 \cdot \begin{bmatrix} 1,5693 & 0 & 0 \\ 0 & -0,1122 & 0 \\ 0 & 0 & -0,1122 \end{bmatrix} = \begin{bmatrix} 11,623 & 0 & 0 \\ 0 & -0,831 & 0 \\ 0 & 0 & -0,831 \end{bmatrix} \text{MPa}$$

EX3

load history



$A = 10\ cm^2$

$E = 2 \cdot 10^{11}\ Pa$

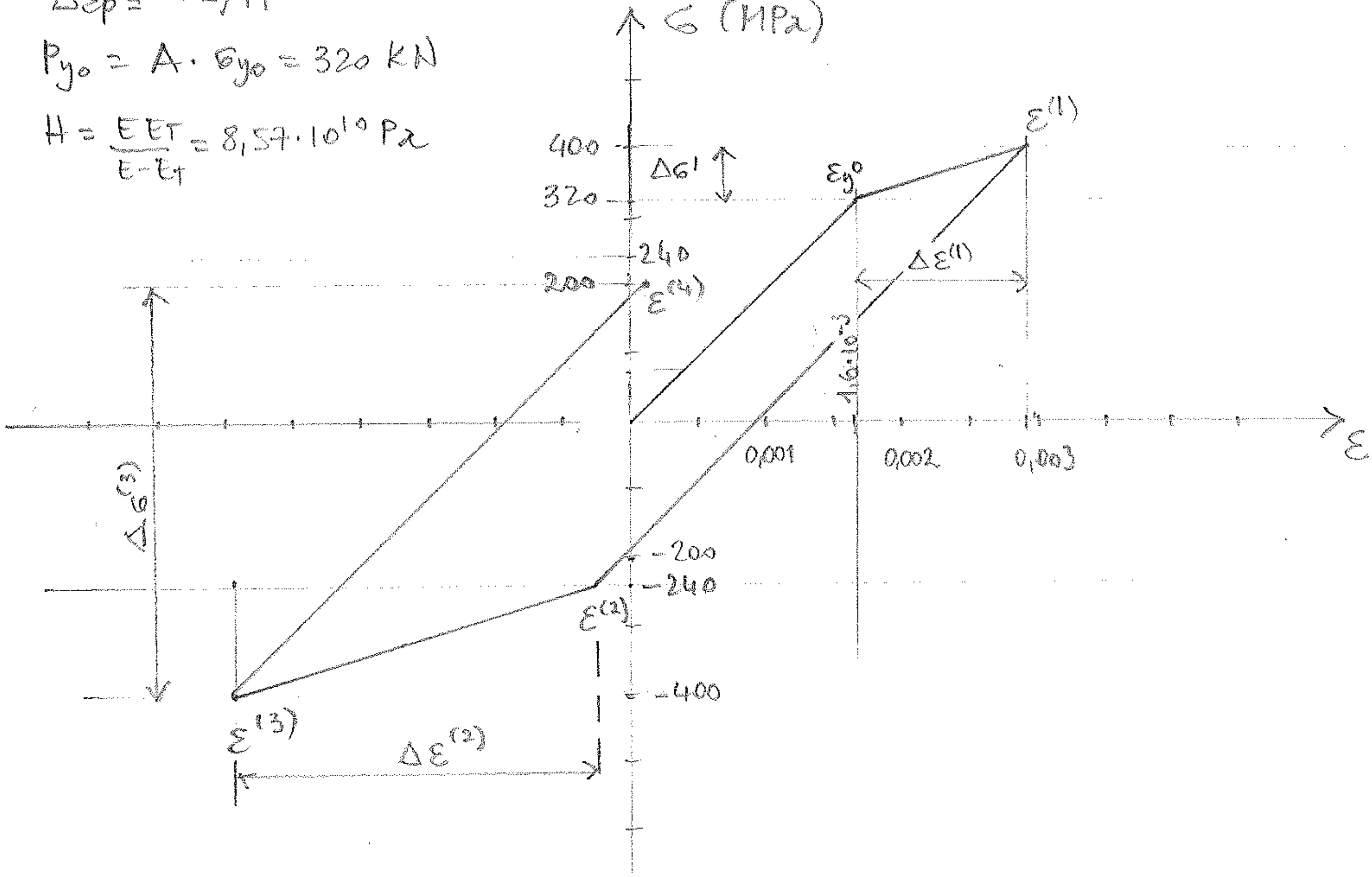
$E_T = 6 \cdot 10^{10}\ Pa$

$\sigma_{y0} = 3{,}2 \cdot 10^{8}\ Pa$

$\Delta\varepsilon_p = \Delta\sigma / H$

$P_{y0} = A \cdot \sigma_{y0} = 320\ kN$

$H = \dfrac{E E_T}{E - E_T} = 8{,}57 \cdot 10^{10}\ Pa$



$P_y^0 = A \cdot \sigma_y^0 = 320\ kN$

$\varepsilon_y^0 = \dfrac{3{,}2 \cdot 10^8}{2 \cdot 10^{11}} = 0{,}0016$

1) $\left.\begin{aligned} \Delta\varepsilon_p^1 &= \dfrac{\Delta\sigma^1}{H} = \dfrac{8 \cdot 10^7}{8{,}57 \cdot 10^{10}} = 0{,}00093 \\ \Delta\varepsilon_e^1 &= \dfrac{\Delta\sigma^1}{E} = \dfrac{8 \cdot 10^7}{2 \cdot 10^{11}} = 0{,}0004 \end{aligned}\right\} \Delta\varepsilon^{(1)} = \Delta\varepsilon_e^1 + \Delta\varepsilon_p^1 = 1{,}33 \cdot 10^{-3}$

$\varepsilon^{(1)} = 1{,}6 \cdot 10^{-3} + 1{,}33 \cdot 10^{-3} = 2{,}93 \cdot 10^{-3}$

$\sigma_y^1 = 4 \cdot 10^8\ Pa$

Because of the Kinematic hardening $\sigma_y^2 = 4\cdot10^8 - 3,2\cdot10^8\cdot 2 = -2,4\cdot10^8$ Pa

2) $\varepsilon^{(2)} = \varepsilon^{(1)} - \frac{\Delta\sigma_0^2}{E} = 2,93\cdot10^{-3} - \frac{6,4\cdot10^8}{2\cdot10^{11}} = -2,7\cdot10^{-4}$

$\Delta\sigma_0^2 = 320$ MPa

$$\left.\begin{aligned} \Delta\varepsilon_p^2 &= -\frac{\Delta\sigma^2}{H} = \frac{-160\cdot10^6}{8,57\cdot10^{10}} = -1,87\cdot10^{-3} \\ \Delta\varepsilon_e^2 &= -\frac{\Delta\sigma^2}{E} = -\frac{160\cdot10^6}{2\cdot10^{11}} = -8\cdot10^{-4} \end{aligned}\right\} \Delta\varepsilon^{(2)} = -2,67\cdot10^{-3}$$

$\varepsilon^{(3)} = \varepsilon^{(2)} + \Delta\varepsilon^{(2)} = -2,7\cdot10^{-4} - 2,67\cdot10^{-3} = -2,94\cdot10^{-3}$

3) $\sigma_y^3 = -4\cdot10^8 + 6,4\cdot10^8 = +2,4\cdot10^8$ Pa

the last step is all elastic being $2\cdot10^8 < 2,4\cdot10^8$ Pa.

$\Delta\sigma^{(3)} = 2,0\cdot10^8 + 4\cdot10^8 = 6\cdot10^8$ Pa

$\Delta\varepsilon^{(3)} = \frac{6\cdot10^8}{2\cdot10^{11}} = 3\cdot10^{-3}$

$\varepsilon^{(4)} = \varepsilon^{(3)} + \Delta\varepsilon^{(3)} = -2,94\cdot10^{-3} + 3\cdot10^{-3} = 0,06\cdot10^{-3}$

Displacement history of the truss tip:

t	ε	$u = \varepsilon\cdot l$
0	0	0
t_1	$2,93\cdot10^{-3}$	2,93 mm
t_2	$-2,94\cdot10^{-3}$	-2,94 mm
t_3	$+0,06\cdot10^{-3}$	+0,06 mm