AFEA 13/6/2017

ex. 2

Green Lagrange def tensor:

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0{,}5 & 0 \\ 0 & 0 & +0{,}5 \end{bmatrix} \qquad E = \frac{1}{2}(C - I) = \frac{1}{2}(F^T F - I) \; ; \; J = \det F = 0{,}25$$

$$F^T F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0{,}25 & 0 \\ 0 & 0 & 0{,}25 \end{bmatrix} \qquad E = \frac{1}{2}\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0{,}25 & 0 \\ 0 & 0 & 0{,}25 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

$$= \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0{,}75 & 0 \\ 0 & 0 & -0{,}75 \end{bmatrix} \; ; \; \operatorname{tr} E = -\frac{1{,}5}{2}$$

2nd Piola Kirchhoff stress

$$S = \mathbb{C} E = I \lambda \operatorname{tr} E + 2\mu E = -\frac{1{,}5}{2} \cdot 1{,}15 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 2 \cdot 0{,}77 \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{0{,}75}{2} & 0 \\ 0 & 0 & -\frac{0{,}75}{2} \end{bmatrix}$$

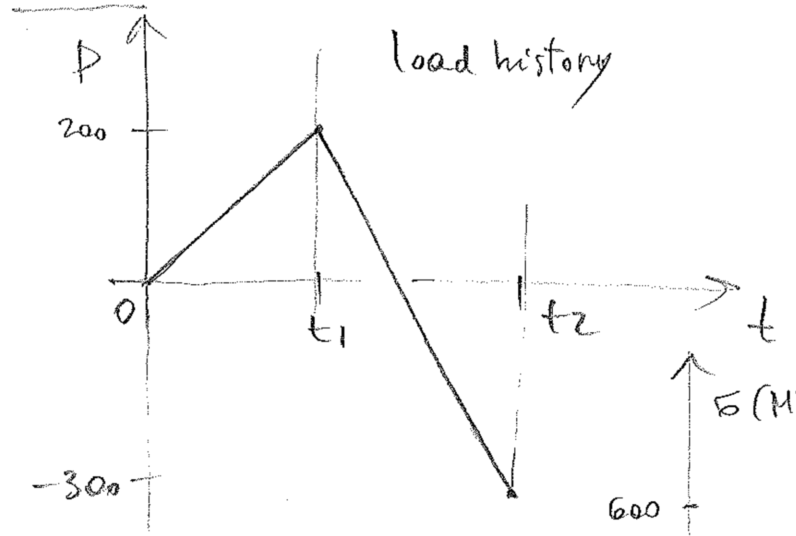
$$S = \begin{bmatrix} -\frac{1{,}725}{2} & 0 & 0 \\ 0 & -\frac{1{,}725}{2} & 0 \\ 0 & 0 & -\frac{1{,}725}{2} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0{,}5775 & 0 \\ 0 & 0 & -0{,}5775 \end{bmatrix} = \begin{bmatrix} -0{,}862 & 0 & 0 \\ 0 & -1{,}44 & 0 \\ 0 & 0 & -1{,}44 \end{bmatrix} \text{MPa}$$

Cauchy stress

$$\mathbf{P} = FS \qquad P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0{,}5 & 0 \\ 0 & 0 & 0{,}5 \end{bmatrix} \cdot \begin{bmatrix} -0{,}862 & 0 & 0 \\ 0 & -1{,}44 & 0 \\ 0 & 0 & -1{,}44 \end{bmatrix} = \begin{bmatrix} -0{,}862 & 0 & 0 \\ 0 & -0{,}77 & 0 \\ 0 & 0 & -0{,}77 \end{bmatrix}$$

$$\boldsymbol{\sigma} = J^{-1} \mathbf{P} F^T = \frac{1}{0{,}25} \cdot \begin{bmatrix} -0{,}862 & 0 & 0 \\ 0 & -0{,}77 & 0 \\ 0 & 0 & -0{,}77 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0{,}5 & 0 \\ 0 & 0 & 0{,}5 \end{bmatrix} = \begin{bmatrix} -3{,}448 & 0 & 0 \\ 0 & -1{,}54 & 0 \\ 0 & 0 & -1{,}54 \end{bmatrix}$$

EX.3



$A = 5\ cm^2$

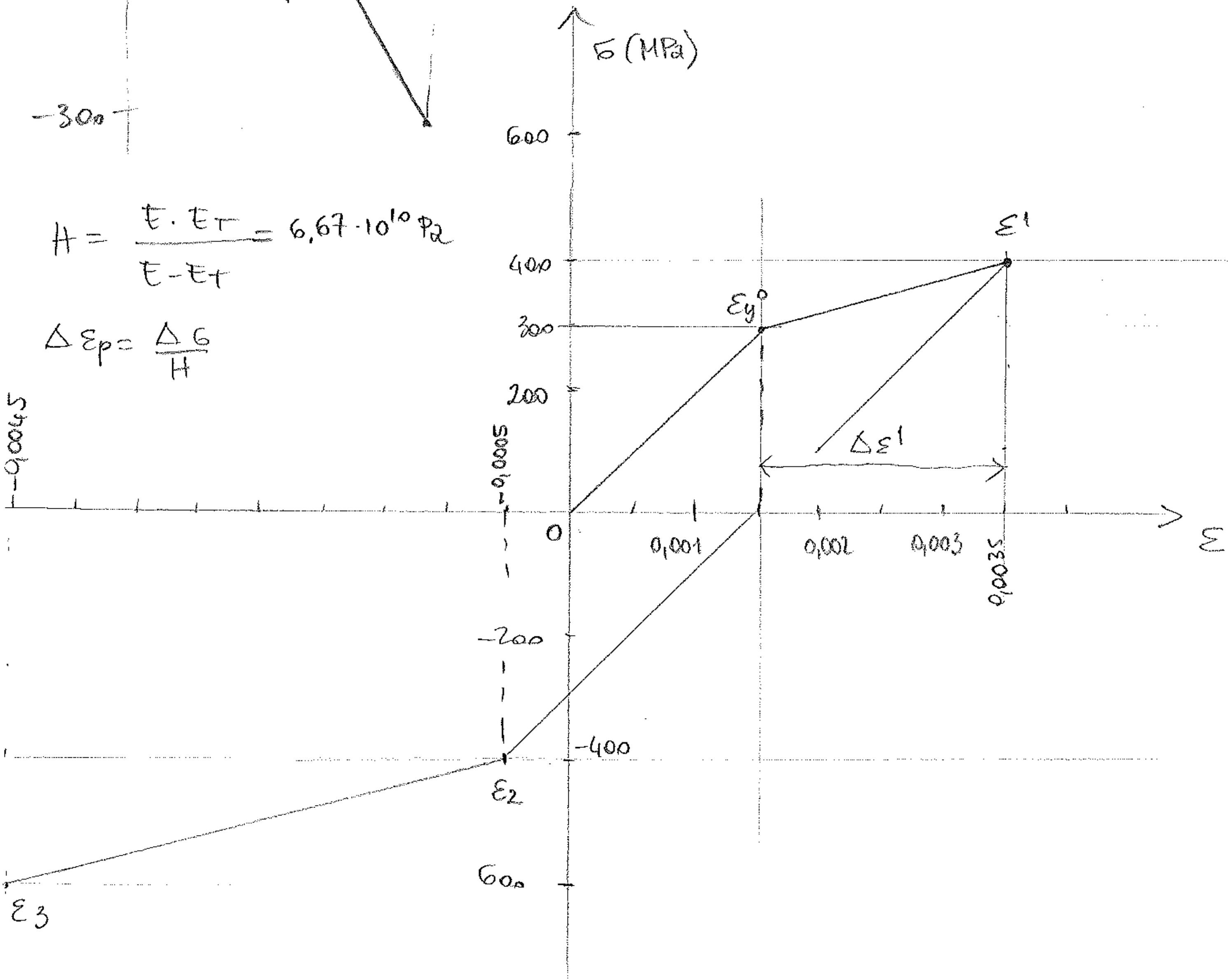
$E = 2 \cdot 10^{11}$ Pa

$E_T = 5 \cdot 10^{10}$ Pa

$\sigma_{y0} = 3 \cdot 10^8$ Pa

$$H = \frac{E \cdot E_T}{E - E_T} = 6{,}67 \cdot 10^{10}\ \text{Pa}$$

$$\Delta \varepsilon_p = \frac{\Delta \sigma}{H}$$



$P_y^0 = A \cdot \sigma_{y0} = 150$ kN

$$\varepsilon_y^0 = \frac{3 \cdot 10^8}{2 \cdot 10^{11}} = 0{,}0015$$

1) $$\left.\begin{aligned} \Delta \varepsilon_p^1 &= \frac{\Delta \sigma^1}{H} = \frac{1 \cdot 10^8}{6{,}67 \cdot 10^{10}} = 0{,}0015 \\ \Delta \varepsilon_e^1 &= \frac{\Delta \sigma^1}{E} = \frac{10^8}{2 \cdot 10^{11}} = 0{,}0005 \end{aligned}\right\} \Delta \varepsilon^1 = \Delta \varepsilon_e^1 + \Delta \varepsilon_p^1 = 0{,}002$$

$\sigma_{y1} = 4 \cdot 10^8$ Pa

2) $$\varepsilon_2 = \varepsilon^1 - \frac{\Delta \sigma_0^2}{E} = 0{,}0035 - \frac{8 \cdot 10^8}{2 \cdot 10^{11}} = 0{,}0035 - 0{,}004 = -0{,}0005$$

$$\left.\begin{aligned} \Delta \varepsilon_p^2 &= \frac{-\Delta\sigma^2}{H} = \frac{2\cdot 10^8}{6{,}67\cdot 10^{10}} = -0{,}003 \\ \Delta \varepsilon_e^2 &= \frac{-\Delta\sigma^2}{E} = \frac{2\cdot 10^8}{2\cdot 10^{11}} = -0{,}001 \end{aligned}\right\} \Delta\varepsilon^2 = \Delta\varepsilon_e^2 + \Delta\varepsilon_p^2 = -0{,}004$$

$$\varepsilon^3 = \varepsilon^2 + \Delta\varepsilon^2 = -0{,}0005 - 0{,}004 = -0{,}0045$$

Displacement history

t	u
0	0
t_1	$\varepsilon^1 \cdot l = 0{,}0035$ m
t_2	$\varepsilon^3 \cdot l = -0{,}0045$ m